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ABSTRACT

The theoretically expected amplitude of the associations of background quasars with foreground galaxies as a result of gravitational lensing has been updated in this paper. Since the galactic matter alone yields an amplitude of quasar overdensity smaller than that observed, a special attention has been paid to the examination or re-examination of the uncertainties in the estimate of the quasar enhancement factor arising from the cosmic evolution of galaxies, the core radius and velocity bias of galactic matter distributions, the clusters of galaxies, the obstruction effect by galactic disks, the non-zero cosmological constant, etc. Unfortunately, none of these factors has been shown to be able to significantly improve the situation, although a combination of some effects may provide a result that marginally agrees with observations. It is concluded that the quasar-galaxy association still remains to be an unsolved puzzle in today's astronomy, if the reported quasar-galaxy associations are not due to the statistical variations and/or the observed quasar number counts as a whole have not been seriously contaminated by gravitational lensing.

Subject headings: galaxies: clusters: general — gravitational lensing — quasars: general

1. Introduction

While there are increasing observational evidences for the associations between background quasars and foreground objects such as quasars, galaxies, groups and clusters of galaxies, the theoretical explanations still remain unsatisfactory (see Wu 1996 for a recent review). This has led to the longstanding argument for a non-cosmological origin of quasar redshifts. Nowadays, it is most likely and also widely accepted that the overdensity of high-redshift quasars behind low-redshift objects is relevant to the magnification effect by the gravitational lensing of the foreground objects, though theoretical studies have always found a relatively weak amplitude as compared with observations. In this paper, we intend to update the theoretical estimate of the amplitude of the quasar-galaxy associations in the framework of gravitational lensing, taking into account the influence of various factors such as the galactic matter distributions (e.g. core radius and velocity bias parameter), the galactic morphologies, the galaxy evolution with cosmic epoch, the environmental matter contributions of galaxies from their host clusters, the non-zero cosmological constant, etc. Through the present work we would like to demonstrate how our predictions of the quasar-galaxy associations are affected by these factors. Eventually, we would re-examine the question whether the quasar-galaxy associations can be interpretated as the result of gravitational lensing.

The amplitude of the quasar-galaxy associations is often characterized by the so-called enhancement factor q, which is the ratio of the disturbed or observed surface number density of quasars (galaxies) to the undisturbed or intrinsic value over a given area around galaxies (quasars). Note that the quasar enhancement factor q_q is often used in theoretical studies while observations actually provide the galaxy enhancement factor q_g . We make no distinction below between these two parameters. In 1989 Narayan introduced a simple and

elegant formula in the scenario of gravitational lensing to estimate q:

$$q = \frac{N_q(\langle B + 2.5 \log \mu)}{N_q(\langle B)} \frac{1}{\mu},\tag{1}$$

where N_q is the cumulative quasar number count above the limiting magnitude B, μ is the magnification factor and 2.5 log μ and $1/\mu$ account for the magnification bias and the area distortion because of the gravitational deflection of light, respectively. The advantage of this method is that it is independent of specific lensing models and hence, applicable to various matter distributions. Moreover, it avoids the introduction of the lensing magnification probability and the employment of the quasar luminosity function and therefore, simplifies considerably the theoretical computations. However, one should be cautious of applying this expression for the actual observations: Eq.(1) is valid only for a single galaxy and a given magnification which generally acts as a function of the searching distance from the galaxy. While the measurement of the quasar-galaxy associations is made over a certain area around an ensemble of galaxies or quasars, a statistically expected enhancement factor $\langle q \rangle$ needs to be found in order to compare with observations.

We summarize in Table 1 the updated observations of the quasar-galaxy associations and their resulted enhancement factors. In the present paper, we only concentrate on the optically-selected quasars. It is immediately apparent from Table 1 that observations provide both positive and negative results for the quasar-galaxy associations. In a sense, this is probably representative of the signature of gravitational lensing (Wu 1994). Indeed, previous theoretical studies of the phenomenon in terms of gravitational lensing could give rise to a scenario that is essentially consistent with the major features of the observations if minor modifications to the conventional lensing models were made. For instance, one may achieve the observed enhancement factors by requiring rather a large galaxy velocity dispersion (Webster et al. 1988; Narayan 1989) or rather a steep intrinsic quasar luminosity function (Bartelmann & Schneider 1993). Yet, there are good reasons to believe that

gravitational lensing should be the natural cause for the quasar-galaxy associations, and the inefficiency of the current lensing explanations may arise from our poor understanding of the various effects in modeling of galaxies as lenses.

EDITOR: PLACE TABLE 1 HERE.

We describe the formulas for the estimate of the expected quasar enhancement factor in section 2. In section 3 we investigate the contributions of clusters of galaxies. The numerical computations are carried out in section 4 and our main results are summarized in section 5. Throughout the paper, we adopt a Hubble constant of $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and a flat cosmological model of $\Omega_0 + \lambda_0 = 1$, where Ω_0 and λ_0 denote the matter and cosmological constant contributions, respectively.

2. Galaxies as lenses

It is customary to assume that the contamination of gravitational lensing in the quasar number counts as a whole is negligible. This enables us to use the observed quasar number-magnitude relation $N_q(< B)$ to represent approximately the intrinsic quasar number count in Eq.(1). Numerous quasar surveys using ultraviolet excess and slitless spectroscopy basically yield the similar surface number density of quasars for a given limiting magnitude (Véron 1993; references therein). Our best-fit quasar number-magnitude relation reads

$$N_{q}(\langle B) = \alpha_{1}\beta_{1}10^{(B-19.14)/\beta_{1}}, \qquad (0.2 < z_{s} < 2.2, B < 19.14);$$

$$N_{q}(\langle B) = \alpha_{1}\beta_{1} + \alpha_{2}\beta_{2}(10^{(B-19.14)/\beta_{2}} - 1), \quad (0.2 < z_{s} < 2.2, B > 19.14);$$

$$N_{q}(\langle B) = \alpha_{3}\beta_{3}10^{(B-19.14)/\beta_{3}}, \qquad (2.2 < z_{s} < 3.0, B < 22),$$

$$(2.2 < z_{s} < 3.0, B < 22),$$

where $(\log \alpha_1, \beta_1^{-1}) = (0.62 \pm 0.07, 0.95 \pm 0.04), (\log \alpha_2, \beta_2^{-1}) = (0.61 \mp 0.04, 0.21 \pm 0.03),$ $(\log \alpha_3, \beta_3^{-1}) = (-0.46 \mp 0.02, 0.70 \pm 0.05).$ Since the evaluation of the quasar enhancement factor q from Eq.(1) depends critically on the adopted $N_q(< B)$, we have included the uncertainties in the fit of $N_q(< B)$. Alternatively, we have excluded the variability-selected quasars in order to keep the same selection criteria of quasar samples as those used in the measurements of the quasar-galaxy associations (Table 1). Recall that the variability-selected quasars show a number-magnitude relation without a turnover around $B \approx 19.14$ (Hawkins & Véron 1993), resulting in a relatively small value of q (Wu 1994). Also, it is necessary to note that Eq.(2) is valid within B < 22.

We use two types of density profiles to model galaxy matter distribution: a singular isothermal sphere (SIS) and a softened SIS with a core radius r_c (ISC):

$$\rho = \frac{\sigma_v^2}{2\pi G} \frac{1}{r^2}, \qquad \text{SIS;}$$

$$\rho = \frac{\sigma_v^2}{2\pi G} \frac{1}{r^2 + r_c^2}, \quad \text{ISC,}$$
(3)

in which σ_{DM} is the one-dimensional velocity dispersion of galactic dark matter. Their resulting lensing magnifications are simply

$$\mu = \frac{\theta}{\theta - \theta_E}, \qquad SIS;$$

$$\mu = \left| \left(1 - \theta_E \frac{\sqrt{\theta^2 + \theta_c^2} - \theta_c}{\theta^2} \right) \left(1 + \theta_E \frac{\sqrt{\theta^2 + \theta_c^2} - \theta_c}{\theta^2} - \theta_E \frac{1}{\sqrt{\theta^2 + \theta_c^2}} \right) \right|^{-1}, \quad ISC,$$
(4)

where $\theta_E = 4\pi (\sigma_v/c)^2 (D_{ds}/D_s)$ is the Einstein radius by SIS, $\theta_c \equiv r_c/D_d$, and D_d , and D_d , are the angular diameter distances to the galaxy at redshift z_d , to the background quasar at redshift z_s , and from the galaxy to the quasar, respectively.

To compare with observations, we use the integral form of Eq.(1), i.e., we estimate the average quasar enhancement factor over an area from θ_1 to θ_2 around a foreground galaxy:

$$\overline{q}(B,\theta_1,\theta_2) = \frac{2\int_{\theta_1}^{\theta_2} q_{\text{local}}(B,\theta) \theta d\theta}{\theta_2^2 - \theta_1^2},\tag{5}$$

where q_{local} denotes the "local" value given by Eq.(1). For an ensemble of galaxies as lenses we adopt the Schechter function at $z \approx 0$: $\phi(L,0)dL = \phi_*(L/L_*)^{\alpha} \exp(-L/L_*)dL/L_*$. This expression can be converted into the velocity dispersion distribution through the

empirical formula between galactic luminosity and central velocity dispersion, namely, the Faber-Jackson relation for early-type galaxies (E/S0) $L/L_* = (\sigma_v/\sigma_*)^4$ and the Tully-Fisher relation for spiral galaxies (S) $L/L_* = (\sigma_v/\sigma_*)^{2.6}$, where σ_* is the characteristic velocity dispersion corresponding to an L_* galaxy. The total population of galaxies with $L > L_{min}$ around redshift z_d is thus

$$\langle n_g \rangle = \sum_i \int_{L_{min,i}}^{\infty} \gamma_i \phi_i(L, z_d) dL,$$
 (6)

in which i and γ_i represent, respectively, the i-th morphological type and composition of galaxies. In the following computation, we adopt the parameters (σ_*, L_*, α) given by Efstathiou, Ellis, & Peterson (1988) in B_T band (see also Fukugita & Turner 1991) and the morphological composition E:S0:S=12:19:69 found by Postman & Geller (1984).

We now deal with the spatial distribution of galaxies. The proper distance within redshift dz_d of z_d in an $\Omega_0 + \lambda_0 = 1$ universe is given by

$$dr_{prop,z_d} = \frac{c}{H_0} \frac{dz_d}{(1+z_d)\sqrt{\Omega_0(1+z_d)^3 + 1 - \Omega_0}},$$
(7)

and the angular diameter distance from z_1 to z_2 is thus

$$d(z_1, z_2) = \frac{c}{H_0} \frac{1}{1 + z_2} \int_{z_1}^{z_2} \frac{dz}{\sqrt{\Omega_0 (1 + z)^3 + 1 - \Omega_0}}$$
(8)

so that the angular diameter distances to the foreground galaxy (D_d) , to the background quasar (D_s) and from the galaxy to the quasar (D_{ds}) are $D_d = d(0, z_d)$, $D_s = d(0, z_s)$ and $D_{ds} = d(z_d, z_s)$, respectively.

Because the galaxies in the measurements of quasar-galaxy associations are most likely located at redshifts between ~ 0.1 and ~ 1 , we need to include the effect of galaxy evolution. For simplicity, we consider two types of empirical evolutionary models: (I) the luminosity-dependent evolution by Broadhurst et al. (1988) and (II) the galaxy merging model by Broadhurst et al. (1992). The increase of galaxy number density $\phi_i(L, z_d)$ with

redshift z_d in Model I is given by

$$\log \phi_i(L, z_d) = \log \phi_i(L, 0) + (0.1z_d + 0.2z_d^2) \log \frac{\phi_i(L, 0)}{\phi_i(L_{max}, 0)}, \tag{9}$$

in which L_{max} is the truncated luminosity at $M_{max} = -23.5$, while for Model II,

$$\phi_i(L, z_d) = f(z_d)\phi_i(L, 0),$$

$$f(z_d) = \exp\{-Q[(1 + z_d)^{-\beta} - 1]/\beta\},$$
(10)

where Q describes the galaxy merging rate and β is the ratio of the Hubble constant to the age of the universe. Additionally, the galaxy velocity dispersion in Model II varies with z_d as $\sigma(z_d) = \sigma(0) f(z_d)^{-\nu}$. In the following calculations we choose Q = 4, $\beta = 3/2$ and $\nu = 1/4$ (see also Rix et al. 1994). As a comparison, the velocity dispersion of galaxies in Model I remains unchanged with redshift. This would result in a large population of massive galaxies at high redshifts when coupled with the local Faber-Jackson and the Tully-Fisher relations. So, the effect of gravitational lensing by galaxies predicted in Model I may be overestimated. It should be also noted that both of the evolutionary models employed here are inapplicable to the universe with a nonzero cosmological constant. In general, the introduction of λ_0 would increase the comoving volume and hence reduce the galaxy number density. On the other hand, λ_0 would increase the estimate of the intrinsic luminosity of galaxies. Here, we do not intend to explore an evolutionary model of galaxies for a $\lambda_0 \neq 0$ universe. Instead, we just give a caution that the influence of the galaxy evolution described by Model I and II and the nonzero cosmological constant upon the quasar-galaxy associations can not be simultaneously taken into account.

Noticing that the association areas are very close to the central regions of foreground galaxies in some cases (Table 1), we give a "maximum" estimate of the possible effect of the "obstruction" by the luminous disks of galaxies on the selection of background quasars. To this end, we assume a face-on circular disk with radius R_d for the luminous area of a galaxy. We then require that the search distance around a galaxy should be larger than

 R_d in order to detect the background quasars. For a uniform surface brightness disk with luminosity L, we utilize $R_d = 14(L/L_*)^{1/2}$ kpc to compute the corresponding obstructing size of a galaxy (Grossman & Narayan 1988).

Finally, the expected enhancement factor of the quasar surface number density around foreground galaxies is obtained by

$$\langle q \rangle (B, z_s, \theta_1, \theta_2, m_g) = \frac{\int_0^{z_s} 4\pi D_d^2 (1 + z_d)^3 \langle q n_g \rangle dr_{prop, z_d}}{\int_0^{z_s} 4\pi D_d^2 (1 + z_d)^3 \langle n_g \rangle dr_{prop, z_d}},$$
(11)

where

$$\langle q n_g \rangle = \sum_i \int_{L_{min,i}}^{\infty} \overline{q} \gamma_i \phi_i(L, z_d) dL,$$
 (12)

and m_g denotes the galaxy limiting magnitude used in the searches for the quasar-galaxy associations and is related to the low integration limit L_{min} in Eq.(6) via

$$m_g = M_* - 2.5 \log \frac{L_{min}}{L_*} + K(z_d) + 5 \log \frac{(1+z_d)^2 D_d}{10 \text{pc}}.$$
 (13)

Since the selections of galaxies are often made in the R band, we employ the K correction by Coleman, Wu, & Weedman (1980). Alternatively, we choose intrinsic colors $B_T - R = (1.51, 0.83)$ for (E/S0,S) to transform the luminosity function from B_T band into R band: $M_* = -19.9^{+0.4}_{-0.2} - 1.5 + (R - B_T)$. For background quasars, an approximate color transformation of $B - V \approx 0.4$ is taken as an average value.

3. Cluster contributions

Galaxies are not isolated objects in the universe. We classify galaxies as the cluster populations that trace the gravitational potential of their host clusters and the field populations that follow the large-scale structures of the universe. The contribution of the galaxy environmental matter from large-scale structures to the computation of the quasar enhancement factor has been shown to be negligible (Wu et al. 1996; 1997). Here we focus the effect of cluster matter on $\langle q \rangle$.

For the galaxies bounded in the gravitational potential of their host clusters, cluster matter introduces an asymmetrical matter component superposing on cluster galaxies as lenses. For simplicity, Turner et al (1984) used a uniform matter sheet as the model of a cluster. They inserted this additional mass density into the lensing equation of a galaxy in the study of the multiple images of quasars. This should be a good approximation if the "lensing scale" around the galaxy is much smaller than the cluster size. We essentially follow their methodology by approximating cluster matter contribution as a uniform matter sheet superposed on galaxies. Nevertheless, we utilize a "weighted" mean cluster surface mass density by considering the galaxy distribution in clusters. To this end, we assume an ISC profile with an one-dimensional velocity dispersion σ_c and a core radius r_{cc} for the dark matter distribution of a cluster

$$\Sigma(\zeta) = \frac{\sigma_c^2}{2G} \frac{1}{\sqrt{\zeta^2 + r_{cc}^2}}.$$
(14)

Both the dynamical analysis of the X-ray observations and the study of gravitational arclike images have shown that r_{cc} should be much smaller than the core radius of the cluster luminous matter (X-ray gas and galaxies) (e.g. Wu & Hammer 1993; Durret et al.. 1994). We will take $r_{cc} = 0.1$ Mpc in our computations. A numerical computation shows that our final results remain almost unchanged if r_{cc} varies from 0.05 to 0.25 Mpc. For a cluster galaxy (SIS) at a radius ζ from the cluster center, the magnification becomes then

$$\mu = \frac{\theta}{\theta - \theta_{cE}} \frac{1}{\left(1 - \frac{\Sigma(\zeta)}{\Sigma_{crit}}\right)^2},\tag{15}$$

where θ_{cE} is the Einstein radius and is related to θ_{E} by SIS through $\theta_{cE} = \theta_{E}/(1-\Sigma(\zeta)/\Sigma_{crit})$.

An important parameter appeared in the above equation is the critical surface mass density

$$\Sigma_{crit} = \frac{c^2 D_s}{4\pi G D_d D_{ds}}. (16)$$

It is apparent from Eq.(15) that only those clusters of galaxies whose surface mass densities are close to Σ_{crit} can produce a significant effect. While it is easy to show that Σ_{crit} is

smaller in a λ_0 dominated universe than in an Ω_0 dominated one, clusters of galaxies can act as more efficient lenses if the cosmological constant is nonzero.

We consider two kinds of models for the distribution of galaxies along cluster radius: ISC and King models, which correspond to the following variations of galaxy surface number density $\kappa_g(\zeta)$ with cluster radius:

$$\kappa_g(\zeta) \propto \begin{cases} (\zeta^2 + r_{cg}^2)^{-1/2}, & \text{ISC;} \\ (\zeta^2 + r_{cg}^2)^{-1}, & \text{KING,} \end{cases}$$
(17)

where r_{cg} is the core radius of the galaxy number density profile which has been observationally determined to be $r_{cg} \approx 0.25$ Mpc (Bahcall 1977). Utilizing the same galaxy luminosity distribution and composition as those in the above section, we give the quasar enhancement factor by all the galaxies in clusters with velocity dispersion of σ_c :

$$\langle q \rangle = \frac{\int_0^{z_s} 4\pi D_d^2 (1+z_d)^3 dr_{prop,z_d} \int_0^{R_g} \langle q n_g \rangle \kappa_g(\zeta) 2\pi \zeta d\zeta}{\int_0^{z_s} 4\pi D_d^2 (1+z_d)^3 dr_{prop,z_d} \int_0^{R_g} \langle n_g \rangle \kappa_g(\zeta) 2\pi \zeta d\zeta},$$
(18)

where R_g denotes the cluster radius.

To include the contributions from different clusters of galaxies. we adopt the cluster mass function established by Bahcall & Cen (1993)

$$n_c(>M_c) = \phi_c^* (M_c/M_c^*)^{-1} \exp(-M_c/M_c^*),$$
 (19)

where ϕ_c^* is the normalization and $M_c^* = 3.6 \times 10^{14} M_{\odot}$. M_c refers to the cluster mass within $R_g = 3$ Mpc radius sphere of the cluster center and $M_c = 2\sigma_c^2 R_g/G$ in SIS model. Note that we have used the same R_g for the truncated radius of galaxy distribution as in Eq.(18). The expected quasar enhancement factor around cluster galaxies can be finally written as

$$\langle \langle q \rangle \rangle = \frac{\int_{M_{c,min}}^{\infty} \langle q \rangle (dn_c/dM_c) dM_c}{n_c(>M_{c,min})},$$
(20)

in which $M_{c,min}$ is the low mass limit in the cluster mass function and is taken to be $0.1M_c^*$ below.

4. Numerical results

In order to test the influence of different factors on the prediction of the enhancement factor, we need to construct our "null-hypothesis". We use SIS as the mass density profile for a galaxy which is described by the observed velocity dispersion of stellar population (no bias). Furthermore, galaxies are assumed to be isolated objects in an $\Omega_0 = 1$ universe and non-evolved with cosmic epoch. We will calculate the expected enhancement factor by altering one parameter each time in order to clearly demonstrate its effect on $\langle q \rangle$.

- 1. Galactic morphologies. We first examine how the morphological composition of galaxies affects the estimate of quasar enhancement factor. As is well known, E/S0 galaxies are more massive and hence more efficient lenses than spirals. Thus, a higher quasar enhancement factor is provided by E/S0 galaxies. In Table 2 We have given the theoretically expected enhancement factors by three types of galaxies separately for each measurement. The fraction of E, S0 and S field galaxies remains roughly constant down to a relatively faint magnitude. For instance, even in the deep galaxy surveys to $B \sim 22$ (e.g. Broadhurst et al. 1988) which is comparable to the limiting magnitudes in the searches for the quasar-galaxy associations, one has found a similar galaxy composition to what we have adopted in our null-hypothesis: E:S0:S=12:19:69. However, this composition may vary in clusters which contain more E/S0 galaxies than spirals.
- 2. Cosmic evolution. A question relevant to the morphological composition of galaxies is the galaxy evolutionary effect. The merging model, i.e., our evolution model II motivated by a considerably large population of faint blue galaxies, suggests more galaxies at high redshifts as lenses [Eq.(10)], and almost all of the ellipticals may be the result of galaxy merging at $z \sim 1$. This indeed changes the morphological components. While the velocity dispersion of galaxies was smaller in the past according to the prediction of galaxy mergers, galaxies would appear to be less efficient lenses as compared with our null hypothesis.

Therefore, the evolutionary model in which galaxies merge at recent look-back time (Broadhurst at al. 1992) yields a relatively small quasar enhancement factor. On the other hands, the empirical form of the luminosity-dependent evolution (model I) predicts an increase of $\langle q \rangle$ because of the too many massive galaxies at high redshifts. Again, $\langle q \rangle$ could be significantly overestimated by this unphysical model.

- 3. Core radius. We now turn to the uncertainty in modeling of galaxy matter distributions. Instead of the unphysical condition of an infinite matter density in SIS at the center of a galaxy, ISC is often invoked. The core radius r_c varies from galaxy to galaxy, depending on the galaxy luminosity or velocity dispersion. Turner (1991) and Kochanek (1996) adopted a relation $r_c \sim L^{\delta}$ where δ is determined experientially. Here we utilize a mean core radius of $r_c = 2$ kpc for all the galaxies. Our numerical results (Table 2) indicate that the introduction of a definite core radius of 2 kpc slightly reduces the value of $\langle q \rangle$.
- 4. Velocity bias. Velocity dispersion is a critical parameter in the determination of galaxy masses. It has been argued that the observed velocity dispersion of stars in galaxies may not be representative of the dark matter behavior, i.e., the dark matter may have a larger velocity dispersion σ_{DM} than that observed, and there is a velocity biasing parameter b between the dark matter and the stellar objects. This arises because the stellar population often follows a density profile of r^{-3} while the dark halo exhibits a form of r^{-2} such as SIS and ISC. It has remained unclear whether the bias parameter b should be taken into account in the study of gravitational lensing (Kochanek 1993; 1994). We adopt a value of $b = \sqrt{1.5}$ for E/S0 galaxies (Turner et al. 1984; Fukugita & Turner 1991) to illustrate how our prediction of $\langle q \rangle$ is affected by this uncertainty. As shown in Table 2, the expected $\langle q \rangle$ with and without the bias parameter are indeed different: the correction of velocity dispersion by a factor of b would evidently increase the value of $\langle q \rangle$.
 - 5. Clusters of galaxies. The fraction of all galaxies that belong in clusters or in fields

is quite uncertain, which prevents us from quantitatively setting a plausible mixture of the cluster galaxies and the field galaxies. If the fraction of galaxies in rich clusters is only $\sim 5\%$ (Bahcall 1996), then the cluster contribution to the quasar-galaxy associations may become trivial. Here we discuss an extreme case in which all the galaxies are bounded in clusters. Namely, we evaluate the maximum contribution of the galaxy environmental matter from clusters to the quasar-galaxy associations. Essentially, clusters provide an additional matter component to the galactic lenses and raise the quasar enhancement factor. However, as the mean cluster surface mass density is considerably smaller than the critical value Σ_{crit} for most of the clusters when $\zeta > r_{cc}$, our numerical computations indicate that the environmental matter of cluster galaxies produce little effect on $\langle q \rangle$.

- 6. Obstruction. The obstruction effect by the luminous disk of foreground galaxies turns to be negligible: The largest effect leads to a decrease of $\langle q \rangle$ only by ~ 0.1 for Magain's observation, whereas others are nearly unaffected by obstruction since their searching distances are well beyond the galactic luminous disks.
- 7. Cosmological constant. It appears that a cosmological constant dominant universe of $\lambda_0 = 0.8$ ($\Omega_0 = 0.2$) does not provide a significant difference in the prediction of $\langle q \rangle$ from a matter dominant one ($\lambda_0 = 0$ and $\Omega_0 = 1$), and the increased value of $\langle q \rangle$ by a non-zero λ_0 is rather small (see Table 2). This is consistent with the early analysis by Fukugita et al (1992). The simple reasons are as follows: λ_0 enters into $\langle q \rangle$ through the Einstein radius $\theta_E = 4\pi (\sigma_{DM}/c)^2 (D_{ds}/D_s)$ and the volume element $dV = 4\pi D_d^2 dr_{prop,z_d}$. While $D_{ds}/D_s \approx 1$ for the measurement of quasar-galaxy associations, θ_E is roughly independent of the cosmological models. On the other hand, the contribution of λ_0 is depressed when $\int \langle q n_g \rangle dV / \int \langle n_g \rangle dV$ [Eq.(11)] is employed.
- 8. Quasar number counts. We have not utilized the widely adopted quasar number count in literature, namely, the quasar number-magnitude relation found by Boyle, Shanks

and Peterson (1988). Instead, we have adopted a combination of the quasar number counts by numerous observations. As a consequence, the slope of $d \log N_q/dB$ is somewhat increased at both bright and faint ends of quasar magnitude. We have compared the resulted $\langle q \rangle$ from the Boyle et al. (1988) counts and Eq.(2), and found that the difference is minor. Alternatively, all the results in Table 2 correspond to the quasar redshift limit of $z_s < 2.2$. Noticing that some measurements in Table 1 (e.g. Magain and Van Drom) may contain a large fraction of high redshift ($z_s > 2.2$) quasars, we have also tested the quasar number-magnitude relation $N_q(< B)$ for $2.2 < z_s < 3.0$ [see Eq.(2)]. This leads to a decrease of the prediction of $\langle q \rangle$ because of the flattening of the $N_q(< B)$.

5. Discussion and conclusions

Unlike the previous statistical studies on the quasar-galaxy associations, we have not employed the magnification probability function (e.g. Schneider 1989) in the present paper. This reduces the complexity of computations and avoids the arbitrary choice of a low magnification limit in the convolution of the magnification probability function with the quasar number count or luminosity function. While the quasar enhancement factor around a single galaxy is known, we have statistically obtained the expected quasar enhancement factor $\langle q \rangle$ around foreground galaxies by averaging q over galactic morphologies, luminosities and redshifts. Moreover, we have included the contributions of the environmental matter surrounding galaxies from their host clusters. Other effects such as the possible bias between the velocity dispersion of the stellar population and of the dark matter, the galaxy evolutionary effect and a non-zero cosmological constant have also been considered. As a whole, we have made an extensive theoretical study and have presented an updated estimate of the amplitude of the quasar-galaxy associations in terms of our best knowledge today. Table 2 summarizes the measured and expected values of the quasar enhancement

factors for nine observations.

Overall, as compared with observations, galaxies alone provide a relatively small quasar enhancement factor $\langle q \rangle$. Among various factors and uncertainties we have studied, the following three parameters may produce the most significant effect on the estimates of $\langle q \rangle$: the existence of a velocity bias between the stellar objects and the dark matter, the non-zero cosmological constant and the cluster matter contributions. However, their induced variations in $\langle q \rangle$ are still minor, which cannot increase $\langle q \rangle$ to the values that agree with all the observations. It appears that the combined result of some affects may marginally account for the observed quasar enhancement factor, if a large observational uncertainty is presumed (e.g. Webster's measurement). The results from a combination of a velocity bias b and a non-zero cosmological constant of $\lambda_0 = 0.8$ has been shown in Table 2.

The gravitational lensing mechanism may be able to well reproduce the so-called "null" or negative results of the quasar-galaxy associations, but has less power for the explanation of the large $\langle q \rangle$ events. If the reported quasar-galaxy associations are not due to statistical variations or suffer from other selection effects, this might imply that a steeper quasar number count is required. It is not impossible that the slope of the quasar number-magnitude relation $N_q(< B)$ can be as large as ~ 0.4 at the faint limiting magnitude because there is still sufficient uncertainty in the present quasar number counts. Alternatively, the question remains open whether the observed quasar counts have already been contaminated by lensing. Recall that the gravitational lensing explanation shows a similar inefficiency when applied for the quasar-cluster associations (Wu & Fang 1996).

Yet, the observational selection effects in the measurement of the quasar-galaxy associations are quite complex and error bars have not been given for some of the observations especially for those large enhancement factors. This makes the comparison of the theoretical expectation and the observation very difficult. Because the detections of the

"null" result at the faint quasar limiting magnitude and the positive result at the bright one in the measurements of quasar-galaxy associations are basically consistent with the scenario of gravitational lensing and because it is lack of convincing evidences to support other explanations such as the physical associations, we still believe that the quasar-galaxy associations are relevant to gravitational lensing. However, our detailed examinations of the lensing models indicate that either current measurements are unreliable or we need to modify at least one of the basic hypotheses in the lensing explanation for the quasar-galaxy associations.

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Table 1: Quasar-galaxy associations: observations *

labels	authors	QSO No.	selections	θ range (")	galaxies (R)	observed q_{obs}
С	Crampton	101	V < 18.5, z > 1.5	0 - 6	~ 23	1.4 ± 0.5
K	Kedziora-Chudczer	181	V < 18.5, z > 0.65	6 - 90	~ 21.5	~ 1
M	Magain	153	$\langle V \rangle = 17.4, \langle z \rangle = 2.3$	0 - 3	~ 21	~ 2.8
${ m T}$	Thomas	64	V < 18.5, 1 < z < 2.5	0 - 10	~ 22	1.7 ± 0.4
V	Van Drom	136	$\langle V \rangle = 17.4, \langle z \rangle = 2.3$	3 - 13.7	~ 23	~ 1.46
W	Webster	68	V < 18, 0.7 < z < 2.3	3 - 10	~ 22	~ 2
Y1				2 - 6		1.0 ± 0.3
Y2	Yee	94	V<19,z>1.5	2 - 10	~ 22.5	1.0 ± 0.2
Y3				2 - 15		0.9 ± 0.1

^{*}Data are taken from Narayan (1992) and Wu (1996).

Table 2: Quasar-galaxy associations: expectations

labels	С	K	M	Т	V	W	Y1	Y2	Y3
q_{obs}	1.4 ± 0.5	~ 1	~ 2.8	1.7 ± 0.4	~ 1.46	~ 2	1.0 ± 0.3	1.0 ± 0.2	0.9 ± 0.1
(1)	$1.20^{+0.04}_{-0.04}$	$1.02^{+0.00}_{-0.01}$	$1.97^{+0.24}_{-0.25}$	$1.15^{+0.03}_{-0.03}$	$1.08^{+0.02}_{-0.02}$	$1.12^{+0.04}_{-0.03}$	$1.07^{+0.01}_{-0.01}$	$1.05^{+0.01}_{-0.01}$	$1.03^{+0.01}_{-0.00}$
(2)	$1.26^{+0.06}_{-0.05}$	$1.02^{+0.01}_{-0.00}$	$2.20_{-0.32}^{+0.31}$	$1.18^{+0.05}_{-0.04}$	$1.10^{+0.04}_{-0.02}$	$1.16_{-0.05}^{+0.04}$	$1.08^{+0.02}_{-0.02}$	$1.06^{+0.01}_{-0.02}$	$1.04_{-0.01}^{+0.01}$
(3)	$1.22^{+0.05}_{-0.06}$	$1.02^{+0.00}_{-0.01}$	$1.98^{+0.29}_{-0.18}$	$1.15^{+0.04}_{-0.04}$	$1.09^{+0.02}_{-0.03}$	$1.13^{+0.04}_{-0.04}$	$1.07^{+0.02}_{-0.02}$	$1.05^{+0.01}_{-0.01}$	$1.04_{-0.00}^{+0.01}$
(4)	$1.16^{+0.03}_{-0.03}$	$1.01^{+0.01}_{-0.00}$	$1.74^{+0.13}_{-0.17}$	$1.12^{+0.02}_{-0.02}$	$1.06^{+0.01}_{-0.01}$	$1.10^{+0.01}_{-0.02}$	$1.06^{+0.00}_{-0.01}$	$1.04^{+0.00}_{-0.01}$	$1.03_{-0.01}^{+0.00}$
(5)	$1.21^{+0.03}_{-0.04}$	$1.02^{+0.00}_{-0.01}$	$2.01_{-0.26}^{+0.25}$	$1.16^{+0.02}_{-0.04}$	$1.08^{+0.02}_{-0.01}$	$1.13_{-0.03}^{+0.03}$	$1.07^{+0.01}_{-0.01}$	$1.05^{+0.01}_{-0.01}$	$1.03_{-0.00}^{+0.01}$
(6)	$1.12^{+0.02}_{-0.03}$	$1.01^{+0.00}_{-0.00}$	$1.60^{+0.16}_{-0.17}$	$1.09^{+0.01}_{-0.03}$	$1.04^{+0.01}_{-0.01}$	$1.07^{+0.02}_{-0.02}$	$1.04^{+0.01}_{-0.01}$	$1.03^{+0.01}_{-0.01}$	$1.04^{+0.01}_{-0.01}$
(7)	$1.19_{-0.05}^{+0.04}$	$1.02^{+0.00}_{-0.01}$	$1.71_{-0.19}^{+0.17}$	$1.14^{+0.03}_{-0.04}$	$1.08^{+0.02}_{-0.02}$	$1.12^{+0.03}_{-0.03}$	$1.07^{+0.01}_{-0.01}$	$1.05^{+0.01}_{-0.01}$	$1.03^{+0.01}_{-0.00}$
(8)	$1.26^{+0.05}_{-0.05}$	$1.02^{+0.01}_{-0.00}$	$2.35^{+0.20}_{-0.31}$	$1.20^{+0.04}_{-0.04}$	$1.12^{+0.03}_{-0.04}$	$1.18^{+0.05}_{-0.05}$	$1.08^{+0.01}_{-0.02}$	$1.06^{+0.01}_{-0.01}$	$1.04^{+0.01}_{-0.00}$
(9)	$1.23^{+0.04}_{-0.05}$	$1.04^{+0.02}_{-0.03}$	$2.04^{+0.26}_{-0.28}$	$1.18^{+0.06}_{-0.05}$	$1.12^{+0.04}_{-0.05}$	$1.16^{+0.06}_{-0.05}$	$1.07^{+0.02}_{-0.01}$	$1.06^{+0.02}_{-0.01}$	$1.04^{+0.01}_{-0.00}$
(10)	$1.19_{-0.04}^{+0.04}$	$1.02^{+0.00}_{-0.01}$	$1.80^{+0.18}_{-0.22}$	$1.14_{-0.03}^{+0.03}$	$1.08^{+0.02}_{-0.02}$	$1.12^{+0.03}_{-0.03}$	$1.06^{+0.01}_{-0.01}$	$1.04_{-0.00}^{+0.01}$	$1.03^{+0.01}_{-0.00}$
(11)	$1.26^{+0.04}_{-0.06}$	$1.02^{+0.01}_{-0.00}$	$2.20_{-0.31}^{+0.24}$	$1.19^{+0.04}_{-0.04}$	$1.11_{-0.03}^{+0.02}$	$1.16^{+0.04}_{-0.04}$	$1.08^{+0.02}_{-0.01}$	$1.06^{+0.01}_{-0.01}$	$1.04^{+0.01}_{-0.01}$
(12)	$1.33^{+0.04}_{-0.07}$	$1.03^{+0.01}_{-0.01}$	$2.57_{-0.29}^{+0.10}$	$1.25^{+0.04}_{-0.05}$	$1.16^{+0.04}_{-0.05}$	$1.25^{+0.07}_{-0.07}$	$1.10^{+0.01}_{-0.02}$	$1.07^{+0.01}_{-0.01}$	$1.05^{+0.01}_{-0.01}$

¹Null-hypothesis: E:S0:S=12:19:69, SIS model, no velocity bias, no-evolution and $\lambda_0 = 0$. The error bars are the combined result of the uncertainties in the quasar number counts $N_q(S)$, the luminosity function $\phi(L, z)$ and the characteristic velocity dispersion σ_* .

²E galaxies as lenses only.

³S0 galaxies as lenses only.

⁴S galaxies as lenses only.

 $^{^5}$ Evolution model I: the luminosity-dependent evolution.

⁶Evolution model II: the merging model.

⁷ISC with a core radius of $r_c = 2$ kpc.

⁸With a biasing parameter $b = (3/2)^{1/2}$ for velocity dispersion of E/S0 galaxies.

 $^{^9\}mathrm{Environmental}$ effect from cluster matter contribution.

¹⁰Obstruction effect by foreground galactic disks.

 $^{^{11} \}text{The universe}$ with a non-zero cosmological constant of $\lambda_0 = 0.8.$

 $^{^{12}}$ Combined result of a velocity bias b and a non-zero cosmological constant of $\lambda_0=0.8.$

REFERENCES

Bahcall, N. A. 1977, ARA&A, 15, 505

Bahcall, N. A. 1996, astro-ph/9611148

Bahcall, N. A. & Cen, R. 1993, ApJ, 407, L49

Bartelmann, M. & Schneider, P. 1993, A&A, 268, 1

Boyle, R. J., Shanks, T., & Peterson, B. A. 1988, MNRAS, 235, 935

Broadhurst, T. J., Ellis, R. S., & Glazebrook, K. 1992, Nature, 355, 55

Broadhurst, T. J., Ellis, R. S., & Shanks, T. 1988, MNRAS, 235, 827

Coleman, G. D., Wu, C. C., & Weedman, D. W. 1980, ApJS, 43, 393

Durret, F., Gerbal, D., Lachièze-Rey, M., Lima-Neto, G., & Sadat, R. 1994, A&A, 287, 733

Efstathiou, G., Ellis, R. S., & Peterson, B. A. 1988, MNRAS, 232, 431

Fukugita, M, Futamase, T., Kasai, M., & Turner, E. L. 1992, ApJ, 393, 3

Fukugita, M, & Turner, E. L. 1991, MNRAS, 253, 99

Grossman, S. A. & Narayan, R. 1988, ApJ, 324, L37

Hawkins, M. R., & Véron, P. 1993, MNRAS, 260, 202

Kochanek, C. S. 1993, ApJ, 419, 12

Kochanek, C. S. 1994, ApJ, 436, 56

Kochanek, C. S. 1996, ApJ, 466, 638

Narayan, R. 1989, ApJ, 339, L53

Narayan, R. 1992, Gravitational Lenses, R. Kayser, T. Schramm and L. Nieser, Springer-Verlag, p88

Postman, M., & Geller, M. J. 1984, ApJ, 281, 95

Rix, H.-W., Maoz, D., Turner, E. L., & Fukugita, M. 1994, ApJ, 435, 49

Schneider, P. 1989, A&A, 221, 221

Turner, E. L. 1991, ApJ, 397, 345

Turner, E. L., Ostriker, J. P., & Gott III, J. R. 1984, ApJ, 284, 1

Véron, P. 1993, Gravitational Lenses in the Universe, ed. Surdej, et al. Université de Liège, p237

Webster, R. L., Hewett, P. C., Harding, M. E., & Wegner, G. A. 1988, Nature, 336, 358

Wu, X. P. 1994, A&A, 286, 748

Wu, X. P. 1996, Fundam. Cosmic Phys., 17, 1

Wu, X. P. & Fang, L. Z. 1996, ApJ, 461, L5

Wu, X. P., Fang, L. Z., Zhu, Z. H., & Qin, B. 1996, ApJ, 471, 575

Wu, X. P., Zhu, X. H., Jing, Y. P., & Fang, L. Z. 1997, ApJ, in press

Wu, X. P. & Hammer, F. 1993, MNRAS, 262, 187

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